

O. P. JINDAL SCHOOL, SAVITRI NAGAR
Half Yearly Examination - (2023 – 2024)

Class: XI
Subject: Mathematics

MM: 80
Time: 3 Hrs

(Fifteen Minutes Extra will be given for reading the Question Paper.)

General Instructions:

- i. This question paper has 5 sections A-E.
- ii. Section A has 20 MCQs carrying 1 mark each.
- iii. Section B has 5 questions carrying 2 mark each.
- iv. Section C has 6 questions carrying 3 mark each.
- v. Section D has 4 questions carrying 5 mark each.
- vi. Section E has 3 case based integrated units of assessment (04 marks each) with 5 sub- parts of 1 mark each. Students has to answer any four questions.
- vii. All the questions are compulsory.
- viii. Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.

Section A

1. Evaluate $\frac{n!}{(n-r)!}$ when $r = 3$

- a) $n(n-1)$ b) $n(n-1)(n-2)$ c) $n(n+1)(n+2)$ d) $n(n+2)(n+3)$

OR

If ${}^n C_{10} = {}^n C_{12}$, then the value of ${}^n C_{22}$ is

- a) 22 b) 1 c) 10 d) 12

2. If $\cos A = \frac{1}{3}$, then value of $\cos 2A$ is

- a) $-\frac{5}{9}$ b) $-\frac{7}{9}$ c) -1 d) 5

3. In a circle of radius 10cm, an arc of length 5cm subtend an angle of

- a) π radian b) $\frac{\pi}{2}$ radian c) $\frac{1}{2}$ radian d) 1 radian

OR

Radian measure of 105° is equal to

- a) $\frac{2\pi}{3}$ b) $\frac{7\pi}{12}$ c) $\frac{\pi}{3}$ d) $\frac{3\pi}{8}$

4. The value of $\frac{1 - \tan^2 15^\circ}{1 + \tan^2 15^\circ}$ is

- a) 1 b) $\sqrt{3}$ c) $\frac{\sqrt{3}}{2}$ d) 2

5. $\cos 42^\circ \cos 12^\circ - \sin 42^\circ \sin 12^\circ = ?$

- a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$ d) 1

6. If $x \neq 1$ and $f(x) = \sin 2x + 3$ then range of $f(x)$ is
 a) $[-3, 3]$ b) $[2, 4]$ c) $[0, 3]$ d) $(1, 3]$

OR

7. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is
 a) 2^{mn} b) $2^{mn} - 1$ c) $2mn$ d) m^n
8. If $(2a + b, 11) = (1, a - 3b)$ then value of a and b is
 a) $a = -2, b = 3$ b) $a = 3, b = -2$ c) $a = 2, b = -3$ d) $a = -3, b = -2$
9. If $f : A \rightarrow R$ and $A = \{-2, -1, 0\}$ and $f(x) = 2x - 3$, then range of f is
 a) $\{7, -5, -3\}$ b) $\{-7, 5, -3\}$ c) $\{-7, -5, 3\}$ d) $\{-7, -5, -3\}$
10. Solution of inequality $3(x - 1) \leq 2(x - 3)$ for real x is
 a) $(-\infty, -3)$ b) $(-\infty, 3]$ c) $(-\infty, -3]$ d) $[3, \infty)$

OR

- If x is a negative integer then the solution set of $-12x > 30$ is
 a) $\{-2, -1\}$ b) $\{\dots, -5, -4, -3\}$ c) $\{\dots, -5, -4, -3, -2\}$ d) $\{-2, -1, 0, 1, 2, \dots\}$

11. $\frac{1}{3-4i}$ is represented in the form $x + iy$ is written as

a) $\frac{4}{25} + i\frac{3}{25}$ b) $-\frac{3}{25} + i\frac{4}{25}$ c) $\frac{3}{25} - i\frac{4}{25}$ d) $\frac{3}{25} + i\frac{4}{25}$

12. If $z = (3 + \sqrt{2}i)$ then $z\bar{z} = ?$

a) 5 b) 7 c) 11 d) $\sqrt{11}$

13. Value of $\sqrt{-25} \times \sqrt{-49}$ is

a) 35 b) -35 c) 35i d) -35i

14. Which of the following is true?

a) $1 - i < 1 + i$ b) $2i + 1 > -2i + 1$ c) $2i > 1$ d) none of these

OR

If $z = i^9 + i^{19}$, then z is equal to

a) $0 + 0i$ b) $1 + 0i$ c) $0 + i$ d) $1 + 2i$

15. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $B = \{2, 3, 5, 7\}$. Then which of the following is true?

a) $A \cap B = A$ b) $A \cap B = B$ c) $A \cap B \subset B$ d) None of these

16. If $A = \{1, 2, 3, 4, 5, 6\}$ then the number of proper subset of A is

a) 63 b) 64 c) 32 d) 31

17. Which of the following is an empty set

a) $\{x : x \text{ is a real number and } x^2 + 2 = 0\}$ b) $\{x : x \text{ is a real number and } x^2 - 2 = 0\}$
 c) $\{x : x \text{ is a real number and } (x - 1)(x - 2)(x - 3) = 0\}$ d) $\{x : x \text{ is a real number and } x^2 - x - 6 = 0\}$

18. The smallest set A such that $A \cup \{i, o\} = \{a, e, i, o, u\}$ is:

a) $\{a, e\}$ b) $\{a, e, i\}$ c) $\{a, e, u\}$ d) $\{a, e, i, o\}$

OR

For any two sets $A - B$ is equal to:

a) $A \cap B'$ b) $A' \cap B$ c) $A \cap B$ d) none of these

DIRECTIONS: In the question number 19 and 20, a statement of **Assertion(A)** is followed by a statement of **Reason(R)**. Choose the correct option

- (A) Both assertion(A) and reason(R) are true and reason(R) is the correct explanation of assertion(A)
- (B) Both assertion(A) and reason(R) are true and reason(R) is not the correct explanation of assertion(A)
- (C) Assertion(A) is true but reason(R) is false
- (D) Assertion(A) is false but reason(R) is true.

19. Assertion(A):- The number of selection of 20 distinct object taken 8 at a time is same as that taken 12 at a time.
Reason(R) :- ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

20. Assertion(A):- The solution set of $4x - 2 \leq 2x + 10$, $x \in W$ is $\{0, 1, 2, 3, 4, 5\}$
Reason(R) :- In number system, the symbol W denotes the set of whole numbers i.e 0, 1, 2, 3, 4

Section B

21. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$, then find $(B \cup C)'$.

22. If $A = \{9, 10, 11, 12, 13\}$ and $f:A \rightarrow N$ be defined by $f(n) =$ the highest prime factor of n, then find the range of f.

OR

If $f(x) = x^2$, then find $\frac{f(1.1) - f(1)}{1.1 - 1}$.

23. Find $\sin x$ and $\tan x$, if $\cos x = -\frac{3}{5}$ and x lies in third quadrant.

24. Find the number of ways in which two black balls and 3 red balls can be selected from a bag containing 5 black and 6 red balls.

OR

How many chords can be drawn through 21 points on a circle?

25. What is the multiplicative inverse of $3 + \sqrt{7}i$?

Section C

26. Draw venn diagram for the following:

- a) $(A - B)'$
- b) A and B are disjoint set
- c) $(A' \cap B')$.

27. Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a function from Z to Z defined by $f(x) = ax + b$, for some integers a,b. Determine a and b.

28. Prove that $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\sin 4x \cos 2x \cos x$.

OR

Prove that $\cot^2 \frac{\pi}{6} + \operatorname{cosec} \frac{5\pi}{6} + 3\tan^2 \frac{\pi}{6} = 6$.

29. Solve the inequality $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$ and show the graph of the solution on number line.

30. It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?

OR

How many words with or without meaning can be formed using all the letters of the word EQUATION at a time so that the vowels and consonant occurs together?

31. If $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + 1} \right|$.

Section D

32. Find the value of $\sin 18^\circ$.

OR

Prove that $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ = \frac{3}{16}$.

33. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find the minimum marks Sunita must obtain in fifth examination to get grade 'A' in the course.

34. A committee of 7 persons has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of:

- i) exactly 3 girls ii) atleast 3 girls iii) atmost 3 girls.

35. Covert the complex number $Z = 1 - \sqrt{3}i$ into polar form.

Section E

36. A submarine is moving in such a way that at a particular moment of time its angle of elevation for two ships, situated at different positions on the surface of water, is α and β respectively. If $\operatorname{cosec} \alpha = \sqrt{3}$ and $\sec \beta = 2$, then answer the following...

i) What will be the value of $\sec \alpha$?

- a) $\sqrt{\frac{2}{3}}$ b) $\sqrt{\frac{3}{2}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{\sqrt{6}}$

ii) What will be the measure of angle β ?

- a) $\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{12}$

iii) What will be the value of $\tan \alpha$?

- a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{\sqrt{6}}$

iv) What will be the value of $\tan \beta$?

- a) $\sqrt{3}$ b) $\sqrt{2}$ c) $\frac{1}{\sqrt{3}}$ d) $\sqrt{\frac{2}{3}}$

v) What will be the value of $\tan(\alpha + \beta)$?

- a) $\sqrt{3} - \sqrt{2}$ b) $\sqrt{6} + 1$ c) $\frac{1 + \sqrt{6}}{\sqrt{2} - \sqrt{3}}$ d) $\frac{1 - \sqrt{6}}{\sqrt{2} + \sqrt{3}}$

37. Members of a family A, B, C, D, E, F and G are to be seated in a row for photo session.

i) Total number of ways sitting arrangement of seven members is

- a) 28 b) 5040 c) 2520 d) None of these

ii) Total number of arrangements so that A and B are at extreme positions are

- a) 120 b) 60 c) 240 d) 360

iii) Total number of arrangements if C is sitting in the middle is

- a) 720 b) 360 c) 120 d) None of these

iv) Total number of arrangement if D and E sit together is

- a) 720 b) 1440 c) 360 d) 540

v) Total number of arrangement if D and E never sits together

- a) 720 b) 360 c) 3600 d) 1800.

38. A beaker contains 640 litres of 8% solution of boric acid. This is to be diluted by adding 2% Boric acid solution to it.

Based on the above data answer the following questions

i) The initial amount of acid in the beaker is _____ litres.

- a) 640 b) 51.2 c) 588.8 d) 512

ii) If x litres of 2% Boric acid solution is added to the beaker, the amount of acid becomes _____ litres.

- a) $51.2 + \frac{2}{100}x$ b) $51.2 + \frac{8}{100}x$ c) $588.8 + \frac{2}{100}x$ d) $588.8 + \frac{8}{100}x$

iii) The amount of water after the mixing is _____ litres.

- a) $51.2 + \frac{2}{100}x$ b) $51.2 + \frac{98}{100}x$ c) $588.8 + \frac{2}{100}x$ d) $588.8 + \frac{98}{100}x$

iv) The initial quantity of water in the beaker is

- a) 640 b) 51.2 c) 588.8 d) 512

v) If the resulting mixture is to be more than 4% but less than 6% boric acid, the range of x is

- a) $320 \leq x \leq 1280$ b) $320 \leq x \leq 640$ c) $640 \leq x \leq 1280$ d) $1280 \leq x \leq 2560$.
